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EFFECT OF ENERGY SUPPLY TO A GAS ON LAMINAR BOUNDARY LAYER SEPARATION

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A decelerated flow in a supersonic boundary layer containing a heat source modeling an electric discharge is studied numerically. Calculations are performed for a wide range of the source power. The possibility of controlling the boundary layer separation is demonstrated. The boundary layer separation on cooled walls is found to occur substantially later than on thermally insulated walls.

Key words: *supersonic gas flow, electric discharge, boundary layer separation, heat transfer.*

The interest in studying the effect of electric discharges on the aerodynamic characteristics of high-velocity gas flows is caused by a number of applications, such as reduction of drag of structural elements of flying vehicles, generation of forces on lifting planes, control of the flow at the inlet entrance of an air-breathing engine, etc. [1]. Within the framework of the gas-dynamic approach, the discharge interaction with a supersonic flow can be described by the model of a heat source, which implies that the specific energy supply to the flow is a known function of coordinates and time [1, 2]. The effect of electrodes on the flow is ignored.

We consider a steady plane supersonic flow around a body with an electric discharge in the laminar boundary layer. In mathematical simulations, the discharge is modeled by a rectangular source of energy. In the absence of external mass forces, the system of equations that describe the motion of a perfect gas has the following form:

$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} = 0, \qquad \rho u\,\frac{\partial u}{\partial x} + \rho v\,\frac{\partial u}{\partial y} = -\frac{dp}{dx} + \frac{\partial}{\partial y}\Big(\mu\,\frac{\partial u}{\partial y}\Big),$$

$$\rho u\,\frac{\partial H}{\partial x} + \rho v\,\frac{\partial H}{\partial y} = \frac{\partial}{\partial y}\Big(\lambda\,\frac{\partial T}{\partial y} + \mu u\,\frac{\partial u}{\partial y}\Big) + \rho Q, \tag{1}$$

$$\rho = \frac{pm}{RT}, \qquad H = I + \frac{u^2}{2}, \qquad \lambda = \frac{\mu c_p}{\mathrm{Pr}}.$$

Here u and v are the projections of the velocity vector onto the orthogonal coordinate axes x (along the body surface) and y (normal to it), ρ is the density, p is the pressure, T is the temperature, I is the enthalpy, $Q = Q(x, y)$ is the specific heat supplied to a given point of the medium in a unit time, m is the molecular weight of the gas, R is the universal gas constant, μ is the dynamic viscosity, c_p is the specific heat of the gas at constant pressure, and Pr is the Prandtl number (two last parameters are assumed to be constant).

The body surface $y = 0$ is subjected to the conditions $u = 0$ and $v = 0$, and also $\partial T/\partial y = 0$ for a thermally insulated wall or $T = T_w$ for an isothermal wall.

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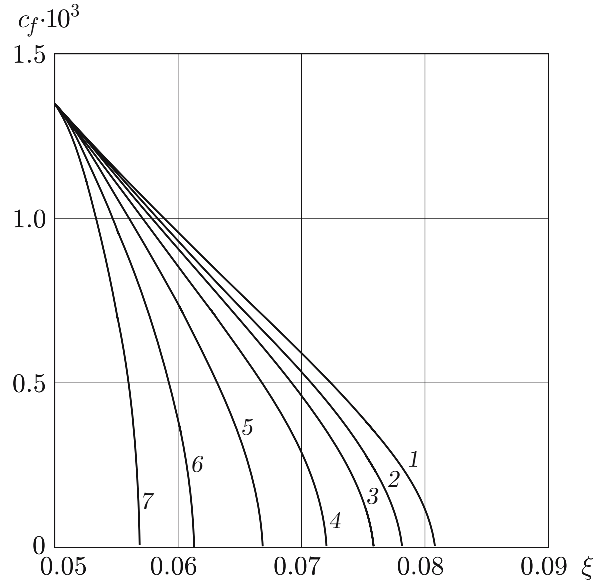

Fig. 1. Distribution of the local skin friction coefficient along a thermally insulated wall for different values of the specific heat supply parameter: $q = 0$ (1), 2 (2), 4 (3), 8 (4), 16 (5), 32 (6), and 64 (7).

The characteristics of the flow on the boundary-layer edge satisfy the equations

$$\rho_e u_e \frac{du_e}{dx} = -\frac{dp}{dx}, \qquad \rho_e u_e \frac{dH_e}{dx} = \rho_e Q_e \tag{2}$$

(the subscript e refers to the quantities on the boundary-layer edge).

System (1) with appropriate boundary conditions is solved numerically by the finite-difference method. An implicit, definitely stable difference scheme is used [3], which ensures the second-order approximation with respect to the spatial grid steps Δx and Δy.

The difference expressions for the derivatives of the sought functions in the streamwise direction have the form

$$\Big(\frac{\partial F}{\partial x}\Big)^j_{i+z} = \beta_z \, \frac{F^j_{i+z} - G_z}{x_{i+1} - x_i},$$

where i and j are the indices of the grid nodes in the directions x and y, $\beta_z = 3$ and $G_z = F^j_i$ for $z = 1/3$, and $\beta_z = 4$ and $G_z = (9F^j_{i+1/3} - 5F^j_i)/4$ for $z = 1$. The solution in the layer $x = x_{i+1}$ is determined from the known values of the functions in the layer $x = x_i$ and includes two stages. At the first stage, the solution is sought at $x = x_{i+1/3}$ $(z = 1/3)$ with an error of the order $O(\Delta x) + O((\Delta y)^2)$. At the second stage, the solution is sought at $x = x_{i+1}$ $(z = 1)$ with the principal part of the approximation error $O((\Delta x)^2) + O((\Delta y)^2)$. Note that the difference scheme described possesses good stabilizing properties [4]. The increase in the boundary-layer thickness is taken into account by consecutive addition of a necessary number of nodes normal to the surface in accordance with the condition of smooth matching [4].

Let us consider the decelerated flow near the stagnation point with a linear distribution of velocity on the boundary-layer edge:

$$u_e = u_0(1 - \xi), \qquad \xi = x/L. \tag{3}$$

This situation is valid, for instance, for an incompressible fluid flow along a flat wall that joins at $x = L$ another wall, which is unbounded and perpendicular to the first wall [5]. The boundary layer in an incompressible fluid with a "single-pitch" velocity profile (3) was studied in [6], where the stream function was presented as a power expansion with respect to ξ and an approximate value of the dimensionless coordinate of the separation point was found: $\xi_s \approx 0.12$.

In these computations, we assume that the Mach number is $\mathrm{M}_0 = 3$, the Prandtl number is $\mathrm{Pr} = 0.72$, the ratio of specific heats is $\gamma = 1.4$, the viscosity μ is temperature-dependent, and the power index is $\omega = 0.76$. External heat supply with a constant dimensionless parameter $q = Q(c_pT_0u_0)^{-1}L$ is provided in a rectangular domain

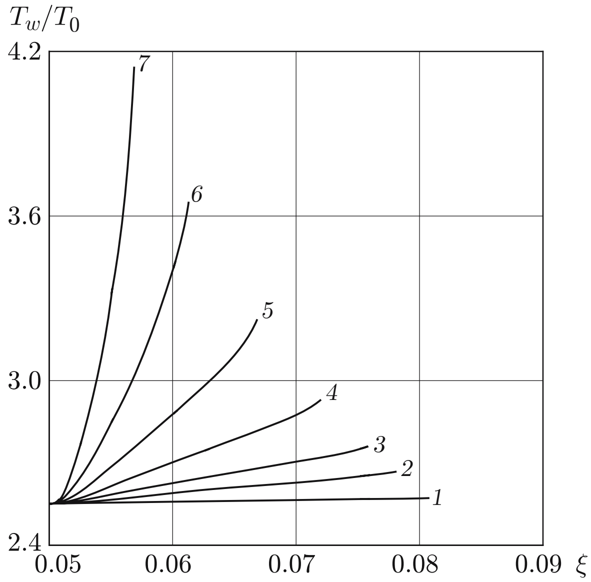

Fig. 2

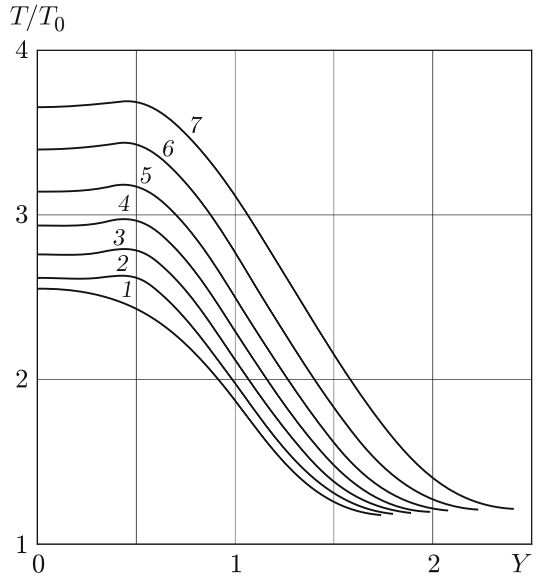

Fig. 3

Fig. 2. Distribution of the near-wall temperature of the gas along a thermally insulated surface for different values of the specific heat supply parameter (notation the same as in Fig. 1).

Fig. 3. Profiles of the gas temperature on a thermally insulated wall ($q = 32$) in different cross sections of the boundary layer: $\xi = 0.050$ (1), 0.052 (2), 0.054 (3), 0.056 (4), 0.058 (5), 0.060 (6), and 0.0613 (7).

$$0.05 < \xi \leqslant 0.10, \qquad 0.4 \leqslant Y \leqslant 1.0,$$
$$Y = y\mathrm{Re}_0^{0.5} L^{-1}, \qquad \mathrm{Re}_0 = \rho_0 u_0 L \mu_0^{-1}, \tag{4}$$

where $f_0 = f_e(0)$. The temperature profiles given below show that this domain is located inside the boundary layer in the cases with both thermally insulated and cooled isothermal walls. Therefore, the pressure $p = p(x)$ at $Q_e = 0$ can be found from Eqs. (2) and (3). The oblique shock wave arising near the separation point is assumed to be weak and to exert no significant influence on the external flow velocity (3).

The initial conditions near the leading edge were determined by solving the corresponding locally self-similar problem in the Lees–Dorodnitsyn variables [7]. The grid used was nonuniform in the direction normal to the body surface, with the nodes being condensed toward the surface. The number of grid nodes in the layer was 150–200, and the longitudinal step in terms of ξ was 0.00001.

Figure 1 shows the distributions of the local skin friction coefficient $c_f = 2\tau_w \rho_e^{-1} u_e^{-2}$ [$\tau_w = (\mu\, \partial u/\partial y)|_{y=0}$ is the friction stress and the Reynolds number is $\mathrm{Re}_0 = 10^6$] along a thermally insulated wall. For $q = 0$, the value of the separation-point coordinate where the skin friction equals zero is smaller than that in the case of an incompressible fluid. Owing to heat release, the streamlines are displaced away from the wall, the boundary layer becomes thicker, and the skin friction decreases. As the parameter q is increased, the separation point is shifted upstream. It should be noted that, as the Reynolds number Re_0 is increased, the curves of the skin friction coefficient similar to the curves plotted in Fig. 1 have a compression coefficient over the ordinate axis, which is proportional to $\mathrm{Re}_0^{-0.5}$. Therefore, when the Reynolds number is changed, the relative coordinates of the separation points remain unchanged as long as the boundary layer remains laminar.

Figure 2 shows the near-wall temperature of the gas as a function of the dimensionless streamwise coordinate. For $q = 64$ (curve 7), the maximum value of the temperature T_w exceeds the corresponding value for $q = 0$ (curve 1) by a factor of 1.6.

Figure 3 shows the temperature profiles in several cross sections of the boundary layer for $q = 32$. It is seen that the gas temperature in each cross section between the wall and the heat-supply region (4) changes insignificantly,

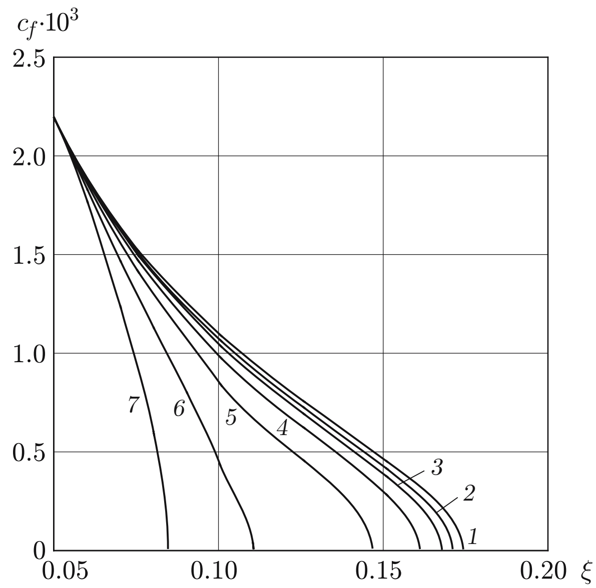

Fig. 4

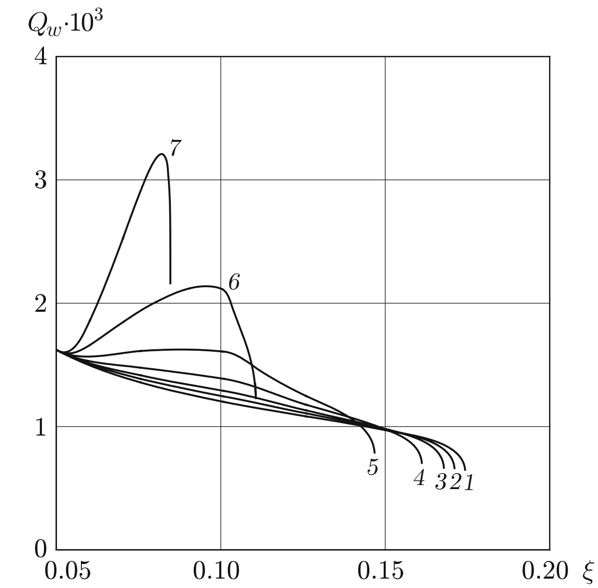

Fig. 5

Fig. 4. Distribution of the local skin friction coefficient along an isothermal wall for different values of the specific heat supply parameter (notation the same as in Fig. 1).

Fig. 5. Distribution of the local heat fluxes along an isothermal wall for different values of the specific heat supply parameter (notation the same as in Fig. 1).

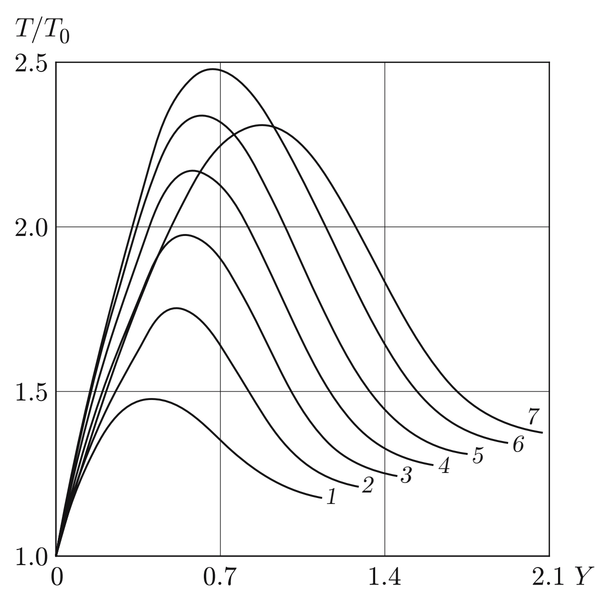

Fig. 6. Profiles of the gas temperature on an isothermal wall ($q = 32$) in different cross sections of the boundary layer: $\xi = 0.05$ (1), 0.06 (2), 0.07 (3), 0.08 (4), 0.09 (5), 0.10 (6), and 0.11 (7).

and the boundary-layer thickness rapidly increases toward the separation point located in an immediate vicinity of the last cross section.

Figure 4 shows the distributions of the skin friction coefficient along a cooled isothermal wall at $T_w = T_0$ and $\mathrm{Re}_0 = 10^6$. It is seen that the value of the coordinate in the absence of heat supply $\xi_s = 0.174$ is substantially greater than in the problem considered by Howarth [6]. For all energy supply intensities, the boundary-layer separation on the cooled wall occurs substantially later than the separation on the thermally insulated wall. Except for the case with $q = 64$, the separation occurs downstream from the heat-supply area (4).

Figure 5 shows the local heat fluxes from the gas to the isothermal wall $Q_w = -2q_w\rho_e^{-1}u_e^{-3}$, where $q_w = -(\lambda\,\partial T/\partial y)\Big|_{y=0}$. For $q = 64$, the maximum heat flux is 2.5 times greater than the corresponding value without heat supply. The heat flux decreases in the vicinity of the separation point, but still remains rather high, in contrast to skin friction.

Figure 6 shows the temperature profiles in the cross sections $\xi = 0.05$, 0.06, 0.07, 0.08, 0.09, 0.10, and 0.11 for $q = 32$. It is seen that the gas in the boundary layer is gradually cooled downstream of domain (4).

Thus, the results of the numerical study of supersonic decelerated flows testify that it is possible to control boundary-layer separation by changing the electric discharge power and by varying the properties of the walls. The boundary-layer separation on cooled isothermal walls is found to occur substantially later than that on thermally insulated walls.

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